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Classification of hermitian forms. VI Group rings

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This paper contains the application of the techniques developed in the preceding papers of the series (listed separately in the references at the end) to calculate groups $L_i(\mathbf{Z}\pi)$ for finite groups π; thus is completed a programme which has occupied the author for a decade. There is no simple formula for the answer, but results are obtained for satisfyingly general classes of groups: we have a reasonable picture for any π with abelian Sylow 2-subgroup, also for π a 2-group.

The contents are listed below. In Chapter 1 we recapitulate definitions and results from earlier papers, and develop some illustrative calculations. Chapter 2 recounts representation theory in the form we need it. We give the calculation in Chapter 3 for abelian 2-groups; then in the long and difficult Chapter 4 for 2-hyperelementary groups with abelian Sylow 2-subgroups. In Chapter 5 we discuss miscellaneous further calculations and problems.

Here is a general summary of our results: First, precise calculations.

(1) $L'(\pi)$ can be computed from knowledge of L groups of hyperelementary subgroups of π, and their mappings (discussion in (2.1), example in (5.3)).

(2) If π is p-hyperelementary with p odd, then (2.4) the only torsion in $L'_r(\pi)$ is $\mathbf{Z}/2$ if $r = 2$ (the usual 'Arf invariant' element) and, for π of even order; (i) orientable case, $\mathbf{Z}/2$ when $r = 3$, mapping isomorphically to $L_3(\pi/\pi^2)$, (ii) nonorientable case, $w({}_2\pi) = 1$, $\mathbf{Z}/2 \oplus \mathbf{Z}/2$ when $r = 3$.

For 2-hyperelementary groups, matters are complicated.

(3) If π is abelian, the torsion in $L_*(\pi)$ is unaltered on replacing π by its Sylow 2-subgroup (2.4.2); it is computed explicitly in (3.3.2) (orientable case) and (3.4.5) and (3.5.1) (nonorientable case). If π merely has abelian Sylow 2-subgroup, the calculation is basically carried out in Section 4 and summarised in (4.7), but we have no explicit formula. See (5.3) for an example, rectifying the announcement in [L]. If π is a dihedral or quaternion 2-group, the L-groups are determined in (5.2).

Next, we have more general results.

(4) $L'_n(\pi)$ is finitely generated; the torsion subgroup has exponent dividing 8 [V]. I know of no example where 8 cannot be replaced by 4.

(5) $L'_{2k+1}(\pi)$ is finite [V]; we have a signature map from $L'_{2k}(\pi)$ with kernel and cokernel finite 2-groups (combine [V] with (2.2.1)). See (5.1) for a precise discussion and conjectures about the image of the signature map.

(6) There are simplifications of the general theory for the case when π is a 2-group (5.2), but again we have no explicit formula.

None of the calculations of Chapter 1 is claimed as original, and a few of the other results have already been obtained by other authors, particularly Bak [4] and Bass [6]. Some discussion of this is given in the final section.

CONTENTS

1. Algebraic L-theory

The purpose of this chapter is to familiarise the reader with our notation and techniques: no really new results will be obtained. In (1.1) we recall from [F], [17] the definitions and basic formal properties of the groups