Lemma. We have
\[ s_{i+j-1}[H_{ij}] = \begin{cases} 
  j, & \text{if } i = 0, \text{ i.e. } H_{ij} = \mathbb{C}P^{j-1}; \\
  2, & \text{if } i = j = 1; \\
  0, & \text{if } i = 1, j > 1; \\
  -(i+j), & \text{if } i > 1.
\end{cases} \]

Proof. Let \( i = 0 \). Since the stably complex structure on \( H_{0j} = \mathbb{C}P^{j-1} \) is determined by the isomorphism \( T(\mathbb{C}P^{j-1}) \oplus \mathbb{C} \cong \bar{\eta} \oplus \ldots \oplus \bar{\eta} \) (j summands) and \( x = c_1(\bar{\eta}) \), we have
\[ s_{j-1}[\mathbb{C}P^{j-1}] = jx^{j-1}(\mathbb{C}P^{j-1}) = j. \]

Now let \( i > 0 \). Then
\[ s_{i+j-1}(T(\mathbb{C}P^i \times \mathbb{C}P^j)) = (i+1)x^{i+j-1} + (j + 1)y^{i+j-1} = \begin{cases} 
  2x^j + (j+1)y^j, & \text{if } i = 1; \\
  0, & \text{if } i > 1.
\end{cases} \]

Denote by \( \nu \) the normal bundle of the embedding \( \iota: H_{ij} \to \mathbb{C}P^i \times \mathbb{C}P^j \). Then
\[ T(H_{ij}) \oplus \nu = \iota^*(T(\mathbb{C}P^i \times \mathbb{C}P^j)). \]

Since \( c_1(\nu) = \iota^*(x + y) \), we obtain \( s_{i+j-1}(\nu) = \iota^*(x + y)^{i+j-1} \).

Assume \( i = 1 \). Then by the previous Proposition,
\[ s_j[H_{1j}] = s_j(T(H_{1j})) = \iota^*(2x^j + (j+1)y^j - (x+y)^j)(H_{1j}) \]
\[ = (2x^j + (j+1)y^j - (x+y)^j)(x+y)(\mathbb{C}P^1 \times \mathbb{C}P^j) \]
\[ = \begin{cases} 
  2, & \text{if } j = 1; \\
  0, & \text{if } j > 1.
\end{cases} \]

Assume now that \( i > 1 \). Then \( s_{i+j-1}(T(\mathbb{C}P^i \times \mathbb{C}P^j)) = 0 \), and by the previous Proposition,
\[ s_{i+j-1}[H_{ij}] = -s_{i+j-1}(\nu)[H_{ij}] = -\iota^*(x + y)^{i+j-1}(H_{ij}) = -(x+y)^{i+j}(\mathbb{C}P^i \times \mathbb{C}P^j) = -\binom{i+j}{i}, \]
which finishes the proof of the Lemma. \( \square \)

Taras Panov
Department of Geometry and Topology
Faculty of Mathematics and Mechanics
Moscow State University, Leninskie Gory
119991 Moscow RUSSIA

E-mail address: tpanov@higeom.math.msu.su