Manifold Atlas: supplementary material for the page **Formal group laws and genera**

Theorem. (Buchstaber).

$$F_U(u,v)=\frac{\sum_{i,j\ge 0}[H_{ij}]u^iv^j}{\left(\sum_{r\ge 0}[\mathbb{C}P^r]u^r\right)\left(\sum_{s\ge 0}[\mathbb{C}P^s]v^s\right)},$$

where H_{ij} $(0\le i\le j)$ are Milnor hypersurfaces and $H_{ji}=H_{ij}$.

Proof. Set $X=\mathbb{C}P^i\times\mathbb{C}P^j$ in Proposition 3.1. Consider the *Poincaré–Atiyah duality* map $D\colon U^2(\mathbb{C}P^i\times\mathbb{C}P^j)\to U_{2(i+j)-2}(\mathbb{C}P^i\times\mathbb{C}P^j)$ and the map $\varepsilon\colon U_*(\mathbb{C}P^i\times\mathbb{C}P^j)\to U_*(pt)=\Omega^U_*$ induced by the projection $\mathbb{C}P^i\times\mathbb{C}P^j\to pt$. Then the composition

$$\varepsilon D\colon U^2(\mathbb{C}P^i\times\mathbb{C}P^j)\to\Omega^U_{2(i+j)-2}$$

takes geometric cobordisms to the bordism classes of the corresponding submanifolds. In particular, $\varepsilon D(u+_{H}v)=[H_{ij}]$, $\varepsilon D(u^kv^l)=[\mathbb{C}P^{i-k}][\mathbb{C}P^{j-l}]$. Applying εD to the equation $u+_{H}v=F_U(u,v)=\sum\alpha_{kl}\,u^kv^l$ of Proposition 3.1 we obtain

$$[H_{ij}]=\sum_{k,\,l}\alpha_{kl}[\mathbb{C}P^{i-k}][\mathbb{C}P^{j-l}].$$

Therefore,

$$\sum_{i,j}[H_{ij}]u^iv^j=\Bigl(\sum_{k,\,l}\alpha_{kl}u^kv^l\Bigr)\Bigl(\sum_{i\ge k}[\mathbb{C}P^{i-k}]u^{i-k}\Bigr)\Bigl(\sum_{j\ge l}[\mathbb{C}P^{j-l}]v^{j-l}\Bigr),$$

which implies the required formula. □

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