Manifold Atlas: supplementary material for the page **Complex bordism**

Proposition. *The geometric cobordism in $\mathbb{C}P^i \times \mathbb{C}P^j$ corresponding to the element $x + y \in H^2(\mathbb{C}P^i \times \mathbb{C}P^j)$ is represented by the submanifold H_{ij}. In particular, the image of the fundamental class $\langle H_{ij}\rangle$ in $H_{2(i+j-1)}(\mathbb{C}P^i \times \mathbb{C}P^j)$ is Poincaré dual to $x + y$.*

Proof. We have $x + y = c_1(p_1^*(\bar\eta) \otimes p_2^*(\bar\eta))$. The classifying map $f_{x+y}\colon \mathbb{C}P^i \times \mathbb{C}P^j \to \mathbb{C}P^\infty$ is the composition of the *Segre embedding*

$$\sigma\colon \mathbb{C}P^i \times \mathbb{C}P^j \to \mathbb{C}P^{ij+i+j},$$
$$(z_0 : \ldots : z_i) \times (w_0 : \ldots : w_j) \mapsto (z_0w_0 : z_0w_1 : \ldots : z_kw_l : \ldots : z_iw_j),$$

and the embedding $\mathbb{C}P^{ij+i+j} \to \mathbb{C}P^\infty$. The codimension 2 submanifold in $\mathbb{C}P^i \times \mathbb{C}P^j$ corresponding to the cohomology class $x+y$ is obtained as the inverse image $\sigma^{-1}(H)$ of a generally positioned hyperplane in $\mathbb{C}P^{ij+i+j}$ (i.e. a hyperplane H transverse to the image of the Segre embedding). By its definition, the Milnor hypersurface is exactly $\sigma^{-1}(H)$ for one of such hyperplanes H. $\square$

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