Manifold Atlas: supplementary material for the page Complex bordism

**Proposition.** The geometric cobordism in $\mathbb{C}P^i \times \mathbb{C}P^j$ corresponding to the element $x + y \in H^2(\mathbb{C}P^i \times \mathbb{C}P^j)$ is represented by the submanifold $H_{ij}$. In particular, the image of the fundamental class $\langle H_{ij} \rangle$ in $H_{2(i+j-1)}(\mathbb{C}P^i \times \mathbb{C}P^j)$ is Poincaré dual to $x + y$.

**Proof.** We have $x + y = c_1(p_1^*(\eta) \otimes p_2^*(\eta))$. The classifying map $f_{x+y}: \mathbb{C}P^i \times \mathbb{C}P^j \to \mathbb{C}P^\infty$ is the composition of the Segre embedding

$$\sigma: \mathbb{C}P^i \times \mathbb{C}P^j \to \mathbb{C}P^{i+j},$$

$$(z_0: \ldots: z_i) \times (w_0: \ldots: w_j) \mapsto (z_0w_0: z_0w_1: \ldots: z_kw_l: \ldots: z_iw_j),$$

and the embedding $\mathbb{C}P^{i+j} \to \mathbb{C}P^\infty$. The codimension 2 submanifold in $\mathbb{C}P^i \times \mathbb{C}P^j$ corresponding to the cohomology class $x + y$ is obtained as the inverse image $\sigma^{-1}(H)$ of a generally positioned hyperplane in $\mathbb{C}P^{i+j}$ (i.e. a hyperplane $H$ transverse to the image of the Segre embedding). By its definition, the Milnor hypersurface is exactly $\sigma^{-1}(H)$ for one of such hyperplanes $H$. □

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