Manifold Atlas: supplementary material for the page **Formal group laws and genera**

Theorem. (Quillen). *The formal group law F_U of geometric cobordisms is universal.*

Proof. Let $\mathcal{F}$ be the universal formal group law over a ring A. Then there is a homomorphism $r\colon A \to \Omega_U$ which takes $\mathcal{F}$ to F_U. The series $\mathcal{F}$, viewed as a formal group law over the ring $A \otimes \mathbb{Q}$, has the universality property for all formal group laws over $\mathbb{Q}$-algebras. Such a formal group law is determined by its logarithm, which is a series with leading term u. It follows that if we write the logarithm of $\mathcal{F}$ as $\sum b_k \frac{u^{k+1}}{k+1}$ then the ring $A \otimes \mathbb{Q}$ is the polynomial ring $\mathbb{Q}[b_1, b_2, \ldots]$. By Theorem 3.3, $r(b_k) = [\mathbb{C}P^k] \in \Omega_U$. Since $\Omega_U \otimes \mathbb{Q} \cong \mathbb{Q}[[\mathbb{C}P^1], [\mathbb{C}P^2], \ldots]$, this implies that $r \otimes \mathbb{Q}$ is an isomorphism.

By the Lazard Theorem the ring A does not have torsion, so r is a monomorphism. On the other hand, Theorem 3.2 implies that the image $r(A)$ contains the bordism classes $[H_{ij}] \in \Omega_U$, $0 \le i \le j$. Since these classes generate the whole ring Ω_U, the map r is onto and thus an isomorphism. □

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